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Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.; G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.; ALOIS F. KOVARIK, Instructor in Mathematics and Science, Decorah Institute, Decorah, Ia.; D. G. DORRANCE, Jr., Camden, N. Y.; ELMER SCHUYLER, United States Naval Academy, Annapolis, Md., and COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville, Tenn.

Let x=the time in hours.

Then, at the end of x hours, the unconsumed portions of the candles will be $\frac{4-x}{4}$ and $\frac{5-x}{5}$ of their respective lengths.

$$\therefore \frac{5-x}{5} \div \frac{4-x}{4} = 3.$$

$\therefore x = 3\frac{7}{11}$ hours, the time required.

COROLLARY. Putting a and b=the respective times of consumption, $a>b$, and n=the ratio of unconsumed lengths, we have $\frac{a-x}{a} \div \frac{b-x}{b} = n$.

$$\therefore x = \frac{ab(n-1)}{an-b}.$$

GRUBER.

Also solved by *W. F. SHAW*, Austin, Tex.

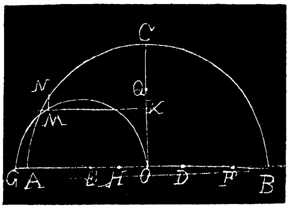
GEOMETRY.

116. **Proposed by P. S. BERG, A. M., Superintendent of Schools, Larimore, N. D.**

Inscribe by rule and compass a regular heptadecagon.

I. **Solution by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.**

The following construction is found on page 95, Vol. IV, of the *Analyst.* This construction is credited to Leybourne's *Mathematical Repository*, 1818.



Let ACB be a semi-circle. Draw the radius CO perpendicular to the diameter AB; on OC and OB take QO equal to the half, and OD equal to the eighth part of the radius; make DE and DF each equal to DQ, and EG and FH, respectively, equal to EQ and FQ; take OK a mean proportional between OH and OQ, and through K draw KM parallel to AB, meeting the semi-circle described on OG in M; and draw MN parallel to OC, cutting the given circle in N. Then the arc AN is the seventeenth part of the whole circumference. The above figure is not drawn to scale.

For a complete and detailed construction see Klein's *Famous Problems in Elementary Geometry*, translated from the German by Professors Beman and Smith, pages 24—41. An analytic solution due to Ampere may be seen in Casey's *Elements of Euclid.* See also THE AMERICAN MATHEMATICAL MONTHLY, Vol. I, page 376, an article by Dr. L. E. Dickson.

II. An approximate construction by **G. B. M. ZERR, A. M., Ph. D.**, Professor of Mathematics and Science, Chester High School, Chester, Pa.

Draw $AB=6\frac{1}{4}$ times the radius. Draw AC. On AC lay off seventeen equal portions. Let AD, DE, EF, and so on, be the equal portions. Join CB and draw DG, EH, FK, etc., parallel to BC. Then AG or GH or HK, etc., are sides of the regular heptadecagon required.

Let a=radius, s=side.

$\therefore\ s=2a\sin\frac{1}{17}\pi=.3675a$.

$6\frac{1}{4}a=6.25a$. $6.25a\div17=.3676a$.

This shows that the above method is a very close approximation.

Prof. Cooper D. Schmitt did not give a construction but gave several references.

117. Proposed by **GUY B. COLLIER**, Schenectady, N. Y.

If $(x',\ y')$ and $(x'',\ y'')$ are the extremities of a pair of conjugate diameters whose eccentric angles are φ' and φ, show that $\varphi'+\varphi=90^\circ$; given $(x'\ y')=(a\sec\varphi',\ b\tan\varphi')$. [From Nichols' *Analytical Geometry.*]

I. Solution by **G. B. M. ZERR, A. M., Ph. D.**, Professor of Mathematics and Science, Chester High School, Chester, Pa., and **B. F. SINE**, Principal of Capon Bridge Normal School, Capon Bridge, W. Va.

Let OQ be the diameter conjugate to OP. Then OQ meets the conjugate hyperbola in Q, R. Let $OA=a$, $OB=b$. Draw PD, QF perpendicular to OD, OF, respectively. Draw DG tangent to the circle with radius OA, FH tangent to the circle with radius OB. Draw OG, OH perpendicular to DG, FH, respectively.

Then $PD=y'$, $OD=x'$, $OF=y''$, $FQ=x''$, $\angle GOD=\varphi'$, $\angle HOA=\varphi$, $x'=a\sec\varphi'$, $y'=b\tan\varphi'$, $x''=a\cot\varphi$, $y''=b\operatorname{cosec}\varphi$. Also $x''=(a/b)y'$, $y''=(b/a)x'$.

$\therefore\ a\cot\varphi=(a/b)b\tan\varphi'$. $\therefore\ \cot\varphi=\tan\varphi'$.

$\therefore\ \varphi=\frac{1}{2}\pi-\varphi'$. $\therefore\ \varphi+\varphi'=\frac{1}{2}\pi$.

II. Solution by **WILLIAM HOOVER, A. M., Ph. D.**, Professsor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

In the last line of the problem, for $b\tan\varphi$ put $b\tan\varphi'$.

It is well known that $x'=a\sec\varphi'$, $y'=b\tan\varphi'$ represents a point on the hyperbola $x^2/a^2-y^2/b^2=1$. Also, $(x'',\ y'')$ is on the conjugate hyperbola, $x^2/a^2-y^2/b^2=-1$, requiring that $x''=a\tan\varphi$, $y''=b\sec\varphi$.

But $x''=(a/b)y'=a\tan\varphi'$, $y''=(b/a)x'=b\sec\varphi$.

$\therefore\ a\tan\varphi'=\pm a\tan\varphi$, and $b\sec\varphi'=b\sec\varphi$, from either of which $\varphi'=\varphi$.

Of a half dozen books on Analytic Geometry, selected somewhat at random, only one explains this matter with a desirable degree of clearness and correctness.

III. Solution by **GEORGE R. DEAN, A. M.**, Professor of Mathematics, University of Missouri, School of Mines and Metallurgy, Rolla, Mo.

Let φ' be the eccentric angle of x', y' with respect to the transverse axis.

φ'', eccentric angle of x'', y'' with respect to transverse axis.

φ''', eccentric angle of x'', y'' with respect to conjugate axis.